Marginally Eternal Inflation

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Abstract

We examine the amount of parameter tuning to make slow-roll inflation marginally eternal by considering an R-invariant chaotic inflation as an example of one-parameter tuning in supergravity. The primordial inflation turns out to be possibly marginal in such a setup.

Realization of slow-roll inflation [1, 2] implies parameter tuning of an inflaton potential in order to make it sufficiently flat. It seems naively that such tuning knows no bounds [3], since the flatter the potential is, the longer the inflation lasts. How flat is the inflaton potential? Is it almost completely flat?

The minimal requirement of inflationary selection [4, 5] is that the potential is so flat as to induce inflation indeed. Observationally, the potential of the primordial inflation should be flat enough to realize several tens of e-folds.

In this note, we examine the amount of parameter tuning to make slow-roll inflation marginally eternal. Namely, we consider marginal inflation, where less tuned potential only realizes non-eternal inflation, or irrelevant inflation, and more tuning leads to relevant inflation.[1]

Let us first present an R-invariant inflation model[2] of the chaotic type as a concrete example of one-parameter tuning [9] in supergravity, which may be of some interest in its own right. The superpotential W and the Kähler potential K of the model is given by

$$W = \tilde{m} X \tilde{\phi} \mathcal{F}(\tilde{\phi}^2), \quad K = |X|^2 + 2A^2 \tilde{x}^2 + 2\tilde{y}^2 + \cdots, \tag{1}$$

where $\tilde{m}$ and A are positive constants,[3] X and $\tilde{\phi}$ denote chiral superfields with $\tilde{\phi} = \tilde{x} + i\tilde{y}$, and $\mathcal{F}(\tilde{\phi}^2) = 1 + \mathcal{O}(\tilde{\phi}^2)$ is a generic holomorphic function of $\tilde{\phi}^2$ in the reduced Planck unit. The R-charges of $\tilde{m}X$ and $\tilde{\phi}$ are two and vanishing, respectively, with their parities odd for a Z_2 symmetry. The ellipsis denotes generic higher-order terms respecting the symmetries.

For the normalized superfield[4] $\phi = \sqrt{A^2+1}\tilde{\phi}$ with $\phi = x + iy$, we obtain

$$W = mX\phi\mathcal{F}\left(\frac{\phi^2}{A^2+1}\right), \quad K = |X|^2 + \frac{2A^2}{A^2+1}x^2 + \frac{2}{A^2+1}y^2 + \cdots, \tag{2}$$

where $m = \tilde{m}/\sqrt{A^2+1}$ and the ellipsis denotes higher-order terms with suppressed variables $x/\sqrt{A^2+1}$ and $y/\sqrt{A^2+1}$ for large A. The kinetic terms are approximately canonical for small $|X|$ and $|\phi|/A$.

[1]The nomenclature is borrowed from that in renormalization theory such as (non-)renormalizable interactions, or (ir)relevant ones.

[2]See Ref.[6, 7]. R-invariance is also utilized in other models [4, 8].

[3] The parameter A is the one to be tuned to realize inflation [9].

[4]The variation of ϕ is as large as $A > 1$ for that of a fundamental variable $|\tilde{\phi}| < 1$.

The potential in supergravity for $\varphi = \sqrt{2}y$ under $x = X = 0$ is given by

$$V = \frac{1}{2}m^2\varphi^2 \left\{ \left| \mathcal{F}\left(\frac{\varphi^2}{2(A^2+1)}\right) \right|^2 \exp\left(\frac{\varphi^2}{A^2+1}\right) + \cdots \right\} = \frac{1}{2}m^2\varphi^2 \left\{ 1 + \mathcal{O}\left(\frac{\varphi^2}{A^2+1}\right) \right\}, \quad (3)$$

which induces chaotic inflation for $|\varphi| \lesssim A$ with large $A \gg 1$. Note that the approximately[5] quadratic form of the inflaton potential is predicted in this framework provided A is sufficiently large.

We now present the condition that such a quadratic potential supports eternal phase [2] at $\varphi = aA \gg 1$ for a positive constant a. The quantum fluctuations are given by

$$\Delta\varphi \simeq \frac{m}{2\pi\sqrt{6}} aA. \quad (4)$$

On the other hand, the slow-roll during the Hubble time is given by

$$\delta\varphi \simeq \frac{2}{aA}. \quad (5)$$

The slow-roll tends to be compensated by the quantum fluctuations for $\Delta\varphi \gtrsim \delta\varphi$, namely,

$$(aA)^2 \gtrsim 4\pi\sqrt{6}m^{-1}. \quad (6)$$

The marginal inflation is realized when this condition is marginally satisfied for the upper bound a of order one to induce inflation.[6]

For the case of primordial chaotic inflation with $m \sim 10^{-5}$, the marginal parameter is given by $aA \sim 10^3$, which implies a well-suppressed φ^4 term in the potential Eq.(3) for several tens of e-folds. If the primordial inflation is marginal, such deviation from the quadratic potential is detectable in principle. Note that the fundamental mass parameter $\tilde{m} = \sqrt{A^2+1}m$ is of order $(100a)^{-1}$, which is rather close to the reduced Planck scale. It might be nearly the largest mass scale in the effective supergravity theory, where the massive degrees of freedom as 'heavy' as the reduced Planck scale have already been integrated out.

[5] If we could set up the superpotential linear in ϕ and the Kähler potential independent of y, we would obtain the quadratic potential as in Ref.[7].

[6] This condition might be regarded to imply large m instead of large A. However, high-energy-scale inflation may be primary [1, 10] rather than primordial under generic multiple inflations [10, 11] in the universe. We here think of $\tilde{m}$ as given and A to be tuned.

Phenomenologically, large kinetic terms seem suitable for not only inflaton fields but also others such as Polonyi [12] and axion fields. We are led to suspect a possibility that those fields reside in a single sector and the choice of a large factor for kinetic terms is due to inflationary reasons.

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